

**ANSWER KEY: SAMPLING DISTRIBUTION OF  $\bar{X}$** 

This answer key provides solutions to the corresponding student activity sheet.

# Sampling Distribution of $\bar{X}$

A data set is not provided for these exercises.

## Exercise 1

(a) What type of distribution can be used to model dragons' wing spans?

**Solution: D. Bimodal**

(b) What type of distribution can be used to model the average of dragons' wing spans?

**Solution: A. Normal.** The Central Limit Theorem tells us that the sample mean is approximately normally distributed regardless of the shape of the original population distribution for such a large sample size as  $n = 100$ .

## Exercise 2

Let  $X_1, X_2, X_3, \dots, X_9$  be independent *normal* random variables with mean  $\mu_X = 3$  and standard deviation  $\sigma_X = 2$ . Let  $\bar{X}$  be the distribution of the mean of these 9 random variables, namely  $\bar{X} = \frac{X_1 + X_2 + \dots + X_9}{9}$ .

(a) What is the shape of the distribution of  $\bar{X}$ ?

**Solution:** Since the parent population is normal, then the distribution of  $\bar{X}$  is **normal**.

(b) What is the mean of the distribution of  $\bar{X}$ ?

**Solution:** The same as the mean of the parent population:  $\mu_{\bar{X}} = 3$ .

(c) What is the standard deviation of the distribution of  $\bar{X}$ ?

**Solution:** It is the fraction  $\frac{1}{\sqrt{9}} = \frac{1}{3}$  of the parent population's standard deviation;  $\sigma_{\bar{X}} = \frac{2}{\sqrt{9}} = 0.667$ .

**(d)** Can we determine  $P(\bar{X} < 2.5)$  using a z-score? You do not need to compute this probability, just answer **yes** or **no** and briefly explain why or why not.

**Solution: Yes.**  $\bar{X}$  is normally distributed and we know the mean and standard deviation of the parent population. We can convert  $x = 2.5$  to a z-score using the mean and standard deviation of  $\bar{X}$ .

### Exercise 3

Let  $X_1, X_2, X_3, \dots, X_{36}$  be independent *skewed* random variables with mean  $\mu_X = 3$  and standard deviation  $\sigma_X = 2$ . Let  $\bar{X}$  be the distribution of the mean of these 36 random variables, namely  $\bar{X} = \frac{X_1 + X_2 + \dots + X_{36}}{36}$ .

**(a)** What is the approximate shape of the distribution of  $\bar{X}$ ?

**Solution:** Most likely, since the sample size is 36, the distribution of  $\bar{X}$  is approximately normal. It may depend though on how "skewed" the  $X_i$  are. Recall that the sample size required for convergence to normality depends on the shape of the original distribution.

**(b)** What is the mean of the distribution of  $\bar{X}$ ?

**Solution:** The same as the mean of the parent population:  $\mu_{\bar{X}} = 3$ .

**(c)** What is the standard deviation of the distribution of  $\bar{X}$ ?

**Solution:** It is the fraction  $\frac{1}{\sqrt{36}} = \frac{1}{6}$  of the parent population's standard deviation;  $\sigma_{\bar{X}} = \frac{2}{\sqrt{36}} = 0.333$ .

**(d)** Can we determine  $P(\bar{X} < 2.5)$  using a z-score? You do not need to compute this probability, just answer **yes** or **no** and briefly explain why or why not.

**Solution: Yes,** if  $\bar{X}$  can be approximated by a normal distribution, which it most likely can be with a sample size of 36, then the probability can be calculated. If the  $X_i$  are highly skewed, then a larger sample size may be required.

## Exercise 4

Let  $X_1, X_2, X_3, \dots, X_{100}$  denote the actual weights of 100 randomly selected bags of sand. The expected weight of each individual bag is  $\mu = 50$  pounds and the standard deviation is  $\sigma = 1$  pound. Let  $\bar{X} = \frac{X_1 + X_2 + \dots + X_{100}}{100}$ .

(a) Assume the bag weights are normally distributed. Randomly select **one** of the 100 bags. What's the probability that it weighs between 49.75 and 50.25 pounds?

**Solution:** The weight  $X$  of a single bag has mean  $\mu = 50$  pounds and standard deviation  $\sigma = 1$  pound. We can use a z-score or Minitab to determine the desired probability:

$$P(49.75 < X < 50.25) = P\left(\frac{49.75 - 50}{1} < Z < \frac{50.25 - 50}{1}\right) = P(-0.25 < Z < 0.25) \cong 0.59871 - 0.40129 \\ = \mathbf{0.19742}$$

(b) Assume the bag weights are normally distributed. What's the probability that the **average weight**  $\bar{X}$  of 100 bags is between 49.75 and 50.25 pounds?

**Solution:** Since the  $X$ 's are normally distributed, then  $\bar{X}$  is normally distributed. The mean and standard deviation of  $\bar{X}$  are  $\mu_{\bar{X}} = 50$  pounds and  $\sigma_{\bar{X}} = \frac{1}{\sqrt{100}} = 0.1$  pound.

$$P(49.75 < X < 50.25) = P\left(\frac{49.75 - 50}{0.1} < Z < \frac{50.25 - 50}{0.1}\right) = P(-2.5 < Z < 2.5) \cong 0.99379 - 0.00621 \\ = \mathbf{0.98758}$$

(c) Assume the bag weights are *positively skewed*. Randomly select **one** of the 100 bags. What's the probability that it weighs between 49.75 and 50.25 pounds?

**Solution:** We cannot determine this since we don't know the distribution of the bag weights.

(d) Assume the bag weights are *positively skewed*. What's the probability that the **average weight**  $\bar{X}$  of 100 bags is between 49.75 and 50.25 pounds?

**Solution:** Since the sample size is 100, then we can assume that  $\bar{X}$  is approximately normally distributed and use the same solution from part (b).

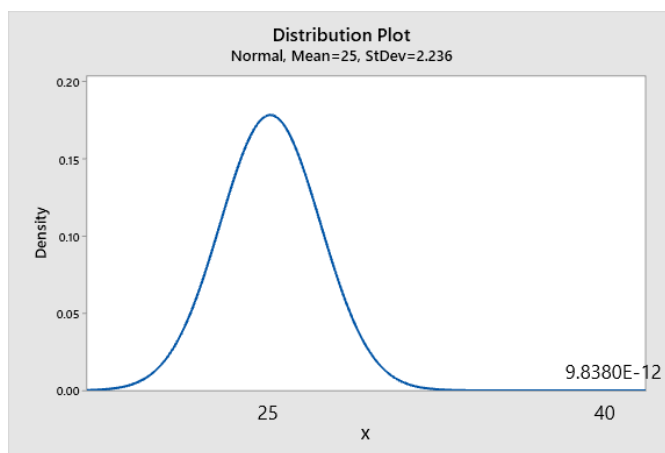
$$P(49.75 < X < 50.25) = P\left(\frac{49.75 - 50}{0.1} < Z < \frac{50.25 - 50}{0.1}\right) = P(-2.5 < Z < 2.5) \cong 0.99379 - 0.00621 \\ = \mathbf{0.98758}$$

## Exercise 5

A small commuter flight leaves from University Park Airport headed to the O'Hare Airport with 20 passengers. By FAA weight standards for carry-on luggage, a passenger's carry-on luggage should not exceed 40 pounds. Let's assume that the weight of a passenger's carry-on luggage is normally distributed with a mean weight of 25 pounds and a standard deviation of 10 pounds, with only 1 carry-on allowed per passenger. What is the probability that the average luggage weight for the 20 passengers exceeds 40 pounds? Report the z-score.

**Solution:** Since the luggage weights are normally distributed, then the mean of the luggage weights  $\bar{X}$  is also normally distributed with mean  $\mu_{\bar{X}} = 25$  pounds and standard deviation  $\sigma_{\bar{X}} = 10/\sqrt{20}$  pounds. The probability is approximately 0:

$$P(\bar{X} > 40) = P\left(Z > \frac{40 - 25}{\frac{10}{\sqrt{20}}}\right) \cong P(Z > 6.71) \cong 0$$



## Exercise 6

Suppose that Beyonce, Jay Z, and Solange go bowling together. Each of them has a bowling score  $X$  that is normally distributed with mean  $\mu = 120$  and standard deviation  $\sigma = 10$ . What is the probability that after bowling one game the average score  $\bar{X}$  for the 3 of them is less than 110?

**Solution:** Since the  $X$ 's are normally distributed, then so is  $\bar{X}$ . The mean and standard deviation of the average score  $\bar{X}$  are 120 and  $10/\sqrt{3} \cong 5.77$ , respectively. In order to determine the standard deviation, we must assume that the three bowling scores are independent of each other. In other words, the score that one gets cannot influence the others.

$$P(\bar{X} < 110) = P\left(Z < \frac{110 - 120}{\frac{10}{\sqrt{3}}}\right) \cong P(Z < -1.73) \cong \mathbf{0.04182}$$